Properties of Arithmetic Progression

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Let S_n denote the sum of n consecutive terms

of an arithmetic progression a_1 , a_2 , a_3 , a_4, ...

Prove that

a) $S_{3n} = 3(S_{2n} - S_n)$;

b) $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2 = \dfrac{n}{2n-1}(a_1^2 - a_{2n}^2)$.

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Let d be difference of the AP.

a) Since $S_{2n} - S_n = \sum_{k=1}^{n} a_{n+k} = \sum_{k=1}^{n}(a_k + nd) = S_n + n^2 d$,

$S_{3n} - S_{2n} = \sum_{k=1}^{n} a_{2n+k} = \sum_{k=1}^{n}(a_k + 2nd) = S_n + 2n^2 d$ then $S_{2n} - 2S_n = n^2 d$,

$S_{3n} - 3S_n = 3n^2 d$. Hence, $S_{3n} - 3S_n = 3(S_{2n} - 2S_n) \Leftrightarrow S_{3n} = 3(S_{2n} - S_n)$.

b) Since $a_{2k-1}^2 - a_{2k}^2 = \left(a_{2k-1} - a_{2k}\right)\left(a_{2k-1} + a_{2k}\right) = -d\left(a_{2k-1} + a_{2k}\right), k = 1,2,..,n$

then $\sum_{k=1}^{n}\left(a_{2k-1}^2 - a_{2k}^2\right) = -d\sum_{k=1}^{n}\left(a_{2k-1} + a_{2k}\right) = -d\sum_{k=1}^{2n} a_k = (-d) \cdot \dfrac{a_1 + a_{2n}}{2} \cdot 2n =$

$\left(-\dfrac{a_{2n} - a_1}{2n-1}\right)(a_1 + a_{2n}) \cdot n = \dfrac{n}{2n-1}(a_1^2 - a_{2n}^2)$.